lar to the line joining the centers of the given circles. This offers no difficulty and the problem is solved.

Also solved by *ELMER SCHUYLER, HALLET E. McCLINTOCK;* and *CHARLES C. CROSS.* Professor Young furnished a neat diagram to accompany his solution of Problem 131.

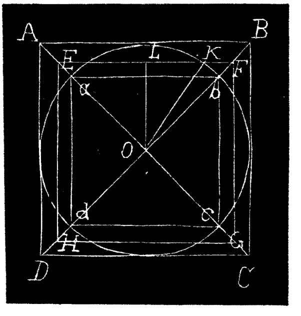
CALCULUS.

99. Proposed by L. C. WALKER, Associate Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

The axis of three equal right circular cylinders intersect at right angles. Find the volume of the solid common to all.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School Chester, Pa.

Let $x^2+z^2=a^2$, $y^2+z^2=a^2$, $x^2+y^2=a^2$ be the equations of the cylinders. The projection of the intersection of the first and second on the plane xy is a square.



From $z=0$, to $z=\frac{1}{2}a\sqrt{2}$ the square decreases from $ABCD$ to $abcd$. The area of the projection common to the three cylinders between $z=0$ and $z=\frac{1}{2}a\sqrt{2}$ is eight times the area $LObKL$. From $z=\frac{1}{2}a\sqrt{2}$ to $z=a$ the common volume is the same as the volume common to the first and second.

Let $\angle LOK=\theta$. Area $EFGH=4xy=4(a^2-z^2)$.

Area $LOK=\frac{1}{2}a.OL\sin\theta=\frac{1}{2}ay\sin\theta$.

Area $KOb=\frac{1}{2}a^2(\frac{1}{4}\pi-\theta)$.

$\tan\theta=LK/OK=x/y=\sqrt{(a^2-y^2)}/y=z/\sqrt{(a^2-z^2)}$;

$\therefore \sin\theta=z/a$. $\therefore$ area $LOK=\frac{1}{2}z\sqrt{(a^2-z^2)}$, area $KOb=\frac{1}{2}a^2(\frac{1}{4}\pi-\sin^{-1}z/a)$.

$$\therefore\ V=8\int_0^{\frac{1}{2}a\sqrt{2}}[z\sqrt{(a^2-z^2)}+a^2(\tfrac{1}{4}\pi-\sin^{-1}z/a)]dz+8\int_{\frac{1}{2}a\sqrt{2}}^{a}(a^2-z^2)dz,$$

$$=8a^3(2-\sqrt{2}).$$

Also solved by *PROFS. ANDEREGG, SHERWOOD,* and *SCHMITT.*

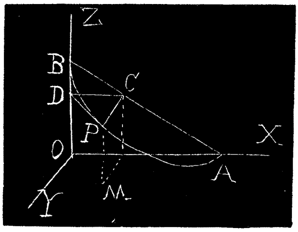
100. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

What is the volume bounded by the surface generated by the circumference of a circle whose diameter is the hypotenuse of a right-angled triangle whose base is b and altitude a, the plane of the circle being perpendicular to the plane of the triangle, the triangle and circle being rigidly connected, and the triangle revolving about its altitude a as an axis ?

Solution by F. ANDEREGG, A.M., Professor of Mathematics, Oberlin College, Oberlin, O., H. C. WHITAKER, Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; and the PROPOSER.

Let $AB=a$, $OA=b$, $AC=\sqrt{(a^2+b^2)}$; the coördinates of P and point in

the given circumference, (x, y, z); and the coördinates of M, the projection of P on the xy plane, (x, y).



Then $DC=bx/a$; $BC=\sqrt{(a^2+b^2)}\dfrac{x}{a}$;

$CA=\dfrac{\sqrt{(a^2+b^2)(a-x)}}{a}$; and $PC=\sqrt{(BC.CA)}$

$$=\frac{\sqrt{[(a^2+b^2)(a-x)x}}{a}.$$

$$\therefore\ DP^2=\frac{b^2x^2+(a^2+b^2)(a-x)x}{a^2}.$$

As the circle with radius DP moves parallel to itself and with its center on AB, it generates the volume required.

$$\therefore\ V=\frac{\pi}{a^2}\int_0^a[b^2x^2+(a^2+b^2)(a-x)x]dx=\frac{\pi a}{6}(a^2+3b^2).$$

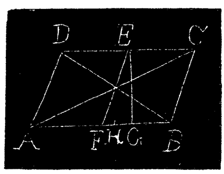
MECHANICS.

97. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

The side AB of the parallelogram $ABCD$ will be a principal axis at the point which divides the distance between the middle point and the foot of the perpendicular from the middle-point of the opposite side in the ratio 2 : 1. The principal moments of inertia about this point are $\frac{1}{3}mb^2\sin^2\beta$, $\frac{1}{36}m(3a^3+4b^2\cos^2\beta)$, where $\beta=\angle A$.

Solution by the PROPOSER.

Let $EH=c$, and let H be the origin, and lines through H parallel to EF, FB axes of coördinates.



$$\therefore\ \Sigma mxy=\rho\sin^2\beta\int_{-\frac{1}{2}a-c}^{\frac{1}{2}a-c}\int_0^b y(x+y\cos\beta)dxdy$$

$=\frac{1}{6}mb\sin\beta(2b\cos\beta-3c)=0$ if HB is a principal axis.

$\therefore\ c=\frac{2}{3}b\cos\beta$. But $FG=b\cos\beta$. $\therefore\ FH:HG=2:1$.

$$\Sigma my^2=\rho\sin^3\beta\int_{-\frac{1}{2}a-c}^{\frac{1}{2}a-c}\int_0^b y2dxdy=\tfrac{1}{3}mb^2\sin^2\beta.$$

$$\Sigma mx^2=\rho\sin\beta\int_{-\frac{1}{2}a-c}^{\frac{1}{2}a-c}\int_0^b(x+y\cos\beta)^2dxdy=\tfrac{1}{12}m(a^2+12c^2-12bc\cos\beta+4b^2\cos^2\beta)$$

$$=\tfrac{1}{36}m(3a^2+4b^2\cos^2\beta).$$